DUOPOLY OF LINUX AND MICROSOFT AS COMPETING SERVER OPERATING SYSTEMS

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ABSTRACT

The paper is motivated by Microsoft Windows and Linux competition at the server operating systems market. Microsoft/Linux mixed duopoly dynamics is analyzed using optimal control mathematical model where Microsoft is considered as a profit-maximizing competitor while Linux as an open source software project is assumed non-for-profit.

Previous studies of mixed duopolies were static except the paper by Cassadeus-Masanell and Ghemawat (2006) where a dynamic mixed duopoly was presented and applied to Windows/Linux competition investigation.

This paper presents some extensions and modifications of Cassadeus-Masanell and Ghemawat's model. Cassadeus-Masanell and Ghemawat assumed that at the demand side of the model in each period of time a new cohort of potential users enters the market, and the size of this cohort does not depend on time. We here assume that the cohort size is increasing exponentially. It is important that the demand side is learning and it affects the cost side. Another addition to the Cassadeus-Masanell and Ghemawat's model is that the cost investments in learning are endogenous.

The conditions when Linux And Windows coexist at the market and when Linux is pushed out by Windows and vice versa are obtained and discussed. The special attention in the model presented is given to a piracy of Windows and strategic contribution to Linux issues which for the first time were discussed also by Cassadeus-Masanell and Ghemawat (2006).

INTRODUCTION

At the moment all the software users are choosing between the three options:

- to buy licenses and use the commercial software (e. g. Microsoft Windows as an operating system, Microsoft Office as an office suite, Microsoft SQL Server as a database server, Microsoft Internet Information Server as a web server, etc.);
- to use free or open source software (e. g. Linux as an operating system, OpenOffice as an office suite, MySQL as a database server, Apache as a web server, etc.);
- to pirate (i. e. to use the commercial software without buying licenses.

These options correspond to three types of software market players:

- profit-maximizers (for example, Microsoft);
- non-for-profit players (for example, Linux team);
- pirates.

In this paper we do not differentiate free and open source software, we refer to these two types of software as non-commercial software.

At the software market the product cost is the sum of fixed cost, vendor profit (margin), and maintenance cost. Fixed costs of commercial and non-commercial software tend to zero, maintenance costs of these two types of software are approximately equal, and the profit of commercial software vendor is positive while one of non-for-profit player is equal to zero. Non-for-profit players indeed earn their money but (opposed to profit-makers) not on sales but on maintenance.

The pirating at the software market is legally prohibited (this prohibition is hardly stimulated by for-profit players, especially by Microsoft). We discuss the software pirating in Soloviev (2008), here we concentrate mostly on the duopoly of commercial and non-commercial software.

-commercial software due to significantly lower cost of ownership (and some other advantages in case of open software where the user can directly affect the product quality). But the situation is much more complex because at the start point of competition between commercial and non-

were occupied by for-profit players and it was very difficult for non-commercial players to distribute their products widely due to high authority of famous commercial products, and almost absolutely no awareness of free software reliability, security, etc. It is known that in 1994 Linux has 0% of server operating systems market, and now Linux has something about 40% of this market (and approximately the same share of market is occupied by Microsoft Windows).

The paper is motivated by Microsoft Windows and Linux competition at the server operating systems market. Microsoft/Linux mixed duopoly dynamics is analyzed using optimal control mathematical model where Microsoft is considered as a profit-maximizing competitor while Linux as an open source software project is assumed non-for-profit.

The history of duopoly studies begins with the works of Cournot (1838), Stackelberg (1934), et al who have studied the symmetric duopolies, while the duopoly of commercial and non-commercial software is asymmetric. In addition, the solutions of Cournot, Stackelberg, Arrow, Nash, et al are static but the main interest at the moment is on the dynamics of software market competition.

The software market players compete in the presence of learning-by-doing (which means that the authority of non-commercial products is growing while the users are learning using these products). The learning-by-doing competition theory initiated by Spence (1981) is focused on the impact of cumulated output on cost reduce.

The prior research of commercial and non-commercial software competition was focused mainly on the customer choice as make-or-buy decision [see, e.g., Kuan (2001)].

Lee and Mendelson (2008) assume that the software market consists of two customer segments with different preferences and is characterized by positive network effects. The commercial player makes product and pricing decisions to maximize its profit. The non-commercial player make product decisions to maximize the weighted sum of the segments' consumer surplus, in addition to their intrinsic motivation.

One of the recent steps in the duopoly theory was to combine the classic market duopoly theory with the demand-side learning and to extend this approach to a dynamic situation where the objectives of players are mixed rather than symmetric. This step was done by Cassadeus-Masanell and Ghemawat (2006) who have presented a dynamic mixed duopoly model and applied this model to Windows/Linux competition dynamics investigation.

This paper presents some extensions and modifications of Cassadeus-

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learning are endogenous.

Using the optimal control theory we obtain the conditions when Linux and Windows coexist at the market and when Linux is pushed out by Windows and vice versa are obtained and discussed. The special attention in the model presented is given to a piracy of Windows and strategic contribution to Linux issues which for the first time were discussed also by Cassadeus-Masanell and Ghemawat (2006).

THE MODEL

The model presented in this paper is modification of Cassadeus-

(2006).

We assume that the operating systems market grow at rate ν. Let $q(t)$ be the portion of new users entering the market at the moment t, who buy Windows, $\rho(t)$ the portion of new users who pirate Windows, $\varepsilon(t)$ the portion of new users who are strategically commited to Linux; each user can either buy Windows, or

pirate Windows, or download Linux, $y_W(t)$ and $y_L(t)$ the cumulative number of Windows and Linux users respectively, w. Then

(1) $$\frac{dy_W(t)}{dt} = \nu q(t),$$

(2) $$\frac{dy_L(t)}{dt} = \nu\big(1-q(t)\big).$$

Let the demand functions of Windows and Linux be linear:

(3) $$p = \alpha_W\left(y_W(t) - sy_L(t)\right)\big(1-q(t)\big)$$

and

(4) $$p = \alpha_L\left(y_W(t) - sy_L(t)\right)\big(1-q(t)\big),$$

here p $s > 0$ is the strength of Linux (which means that the increase of s corresponds to strengthening of Linux due to network externalities), $\alpha_W\left(y_W(t) - sy_L(t)\right)$ and $\alpha_L\left(y_W(t) - sy_L(t)\right)$ are S-shaped technological curves [see Foster (1988)] of Windows and Linux.

When Linux is free and Windows is available at the moment t at price

(5) $$p(t) \geq 0,$$

the portion of customers, precisely indifferent between two operating systems, is given by the next formula:

(6) $$\alpha_W\left(y_W(t) - sy_L(t)\right)\big(1-q(t)-\rho(t)-\varepsilon(t)\big) - p(t) = \alpha_L\left(y_W(t) - sy_L(t)\right)\big(1-q(t)-\rho(t)-\varepsilon(t)\big).$$

Hence

(7) $$p(t) = \alpha_W\left(y_W(t) - sy_L(t)\right)\big(1-q(t)-\rho(t)-\varepsilon(t)\big) - \alpha_L\left(y_W(t) - sy_L(t)\right)\big(1-q(t)-\rho(t)-\varepsilon(t)\big).$$

At the initial moment ($t = 0$) we assume that

(8) $$\alpha_W\left(y_W(0) - sy_L(0)\right) > \alpha_L\left(y_W(0) - sy_L(0)\right).$$

Microsoft is willing to maximize its integral discounted (at the rate δ) profit:

(9) $$J\left(p(t)\right) = \int_0^{+\infty} \nu q(t)\, p(t)\, \mathrm{e}^{-\delta t}\, dt \to \max$$

subject to (1), (2), (5), (7), (8).

RESULTS AND DISCUSSION

The main result of the paper is formulated as the following theorem.

Theorem. Linux and Windows coexist at the market if and only if *s* is greater than 1 and Linux has no strategic commitments with the groups of customers and there is no piracy of Windows. Linux is pushed out by Windows if and only if *s* is less or equal to 1. Windows is pushed out by Linux if and only if *s* is greater than 1 and Linux has a strategic commitments with the groups of customers or there is a piracy of Windows.

The main difference between the model presented in this paper and Cassadeus-Masanell and -
rate is zero presented an example where the piracy of Windows could be beneficial to Microsoft. If we assume that the number of users is growing then we have proven that the piracy of Windows will never bring any benefits to Microsoft.

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